Multivariate Geostatistical Simulation at Red Dog Mine, Alaska, USA

Oy Leuangthong
Department of Civil and Environmental, Engineering, University of Alberta

Terry Hodson
Teck Cominco Limited

Peter Rolley
Teck Cominco Limited

Abstract

Red Dog mine is the world's largest Zn producer. The deposit consists of sulphide ore zones in sedimentary exhalative (sedex) deposits, and is characterized by the presence of multiple metals and multiple ore types. The objective of the case study is to characterize seven different minerals, Zn, Pb, Fe, Ba, sPb, Ag and TOC, within eight different rock types. Geostatistical models were constructed for each variable within the eight rock types, and subsequently assembled to give 40 realizations for six 25ft benches. The stepwise conditional transformation was used to account for the complex multivariate relations. In addition to reproducing the input data, histogram and variogram, the resulting models also respect multivariate relations locally and globally. This paper documents the methodology to construct these models.

A thorough validation procedure was implemented. Comparisons with blasthole data show that the conditional simulations and the long-term models agree in expected value. The main advantages of the conditional simulations are (1) the explicit accounting of multivariate relations between the data in model construction, and (2) the ability to quantify variability and uncertainty at any scale. A small exercise comparing the profit from a kriged and a simulated model is also examined. Results show that the simulation approach showed a 3% increase in profit, relative to the conventional estimation approach.

Introduction

Red Dog mine is located 90 miles north of Kotzebue, Alaska, USA. The property is owned by NANA Corporation, and the mine is operated by Teck Cominco Limited. The deposit consists of sulphide ore zones in sedimentary exhalative (sedex) deposits, and is characterized by the presence of multiple metals and multiple ore types. The mine assays for as many as ten variables; the four primary ones being Zn, Pb, Fe and Ba.

A key issue is the variability within the deposit and the effect of this variability on Zn recovery. Recovery is adversely affected by the presence of high barite and other deleterious minerals and ore textures. The existing long term resource model was constructed by independently kriging the four main variables. Improved multivariate modeling of the different elements and ore types should improve the reliability of the long-term resource model and therefore the prediction of Zn recovery.

The Red Dog Main Pit consists of three geological plates: Upper, Median and Lower. Eight geological rock types were modeled corresponding to four different ore type units in two separate plates. These were chosen because they correspond to a volume that includes both recently mined material and material that will be mined in the near future.

The existing grade models were independently kriged at a 25ft$\times$25ft$\times$25ft resolution. The geostatistical models were simulated at 12.5ft$\times$12.5ft$\times$12.5ft resolution, and were later upscaled to 25ft$\times$25ft$\times$25ft for comparison purposes. There are some good reasons to model at a finer scale than were required. Firstly, the 12.5ft composite data are a good compromise between retaining some of the variability of the smaller drillhole sample data and the faster simulation of larger, and hence fewer blocks. Secondly, the simulation is essentially a "point"-scale simulation; current implementations do not explicitly account for volume-variance relations. Thus, simulating at a finer resolution and then averaging to larger blocks shows the variability of the block grades more accurately.

Six benches were modeled to allow for model reconciliation with blasthole samples. The model limits span a volume that is 4500ft wide (Easting) by 4500ft long (Northing) by 150ft vertical span (Elevation). The model consists of a total of 1,555,200 blocks. The simulations were constructed on a by rock type basis, and all figures shown correspond to one particular rock type. Once all rock types were simulated, the realizations were merged. All global comparisons consist of all rock types taken together.

Available Data

Three types of data were available: drillhole data, composited drillhole data and blasthole data. Multivariate geostatistical modeling considered the 12.5ft composites, while the blasthole data were used to test the predictive ability of the resulting models. A geology model at 25ft resolution was also available. For consistency with the simulation models, the 25ft geology block model was reformatted into a 12.5ft block model.

There were a total of 9847 12.5ft composites available for the eight rock types of interest. The term drillhole (DH) refers to the 12.5ft composites. DH data are at a nominal 100ft×100ft spacing. For these same rock types, there were 58566 blasthole (BH) data available for model validation. BH data are more closely spaced than DH data at 10ft×12ft spacing with 25ft vertical extent (the vertical span is one bench). Figure 1 shows the projection of the available DH data onto the horizontal and vertical plane.

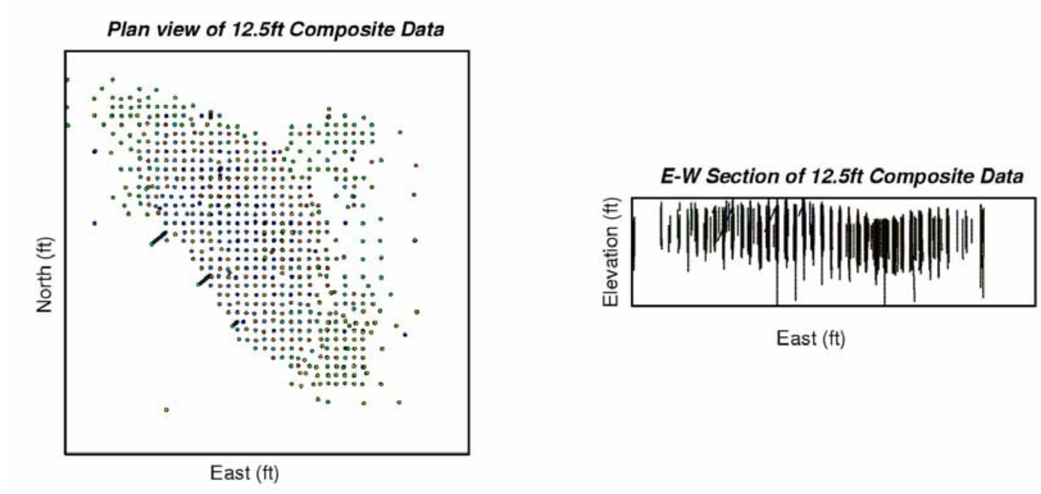

Figure 1 All available drillhole data projected onto a horizontal plane (plan view (left)) and onto a vertical plane (E-W sectional view (right)).

Multivariate Simulation Approach

Conditional simulations were performed for seven variables: Zn, Pb, Fe, Ba, sPb (soluble Pb), Ag, and TOC (total organic content). These seven variables were modeled for each rock type, using Gaussian simulation with stepwise conditionally transformed (Leuangthong, 2003) variables. The main steps of the simulation are:

1. Data declustering to obtain representative distributions for each variable.
2. Transform data in a stepwise conditional (Leuangthong and Deutsch, 2003) manner to obtain independent Gaussian variables.
3. Calculate and model the directional variograms for each of the transformed variables within each rock type.
4. Independently simulate transformed variables via sequential Gaussian simulation (Isaaks, 1990).
5. Back transform simulated values in a stepwise conditional manner to return values to original units.
6. Validation of simulation results to confirm data, histogram and variogram reproduction.
7. Validation using available blasthole data, and compare against existing long term resource model.

Once all variables within all rock types were modeled, the block models were merged to form multiple realizations of the study area for uncertainty assessment and post-processing. All simulation related tasks were performed using GSLIB (Deutsch and Journel, 1998) and other GSLIB-compatible tools.

The proposed methodology is a fairly common approach to geostatistical Gaussian simulation; the main difference is the use of the stepwise conditional transformation (SCT) in place of the conventional normal score transform. SCT is a multivariate Gaussian transformation approach whereby the primary variable is transformed to be standard normal, and all subsequent variables are successively conditioned to the previous variable(s) based on probability binning (Leuangthong, 2003). The transform applies to collocated multivariate data, and facilitates multivariate modeling by removing complex dependencies between the variables, making them independent, prior to simulation. Back transformation restores the complex relationships between the multivariate data.

The need to consider seven variables simultaneously for any one rock type poses a problem in practice; this is the case for Red Dog. The multivariate stepwise conditional transform would require 10^7 composites in order to have a minimum of 10 data per probability class. This is impractical. A *nested* application of the stepwise conditional transformation is proposed to overcome this problem. Accounting for a lower-dimensional multivariate distribution was considered. Inference of a trivariate distribution would require approximately 10^3 or 1000 data to define the conditional distributions with a minimum of 10 data. This is more reasonable given the number of composites available.

The transformation ordering for the stepwise conditional transform will affect the reproduction of the variogram from simulation. Thus, the most important variable or the most continuous variable should be chosen as the primary variable (Leuangthong and Deutsch, 2003). For Red Dog, Zn is the most important variable, and so all others will be conditioned to it. To account for the other six variables, the following sets of transformations were proposed (Table 1):

TABLE 1 Transformation Ordering for Stepwise Conditional Transformation.

Transform No.	Variable	Conditioning Variable(s)
1	Zn	
2	Pb	Zn
3	Fe	Zn, Pb
4	Ba	Zn, Fe
5	sPb	Zn, Pb
6	Ag	Zn, Pb
7	TOC	Zn, Fe

The transformation order reflects the significance Teck Cominco staff attribute to each variable. Zn was considered to be the most important, and so all other variables were transformed conditional to Zn. In all cases, Fe or Pb act as secondary variables, and all remaining variables were then transformed conditional to either Zn and Pb or Zn and Fe.

Declustering was performed to assemble representative distributions for each variable. Given the multivariate nature of this dataset and the intended application of a multivariate transformation technique, declustering must be consistent between all variables. The representative distribution of Zn was obtained by using the accumulated weights obtained from kriging within a rock type; this approach not only respects the rock type being populated, but it also respects the spatial variability of the data and hence their area of influence within this rock type. Secondary variables (say Pb) were declustered through a bivariate calibration of the Pb distribution using both the representative distribution of Zn and the crossplot of Zn and Pb. For all tertiary variables, the same rationale was applicable, and the representative histograms for Fe through TOC were determined using the representative histograms for the two dependent variables plus the trivariate calibration data.

Stepwise conditional transformation was then performed on these representative distributions. Figure 2 shows the scatterplots of the variables resulting from the first transform sequence of Zn, Pb and Fe (see Table 1). The transformed variables are independent and multiGaussian, which translates to a circular shape in the crossplot. From Figure 2, the crossplot between the first two variables (Zn and Pb) appears approximately circular. Crossplots with the third variable (Fe, in this case) show some banding; however this is simply a numerical artefact of having many classes and consequently fewer data within each class (Leuangthong, 2003). Independence of the transformed variables means that each variable can be simulated independently.

Variograms were then calculated and modeled for each of the transformed variables. *Sequential Gaussian simulation* was independently performed for the seven transformed variables on a by rock type basis. A total of 40 realizations were generated for each variable within each rock type. For greater computational efficiency, only those blocks belonging to the specific rock type were simulated. Each realization was then *back transformed* to the original units of the data. Similar to the forward transformation that relied on conditioning one variable to another, the back transformation for each simulated realization must be performed in a conditional fashion. For example, the back transform of Fe is conditional to the simulated values for Zn and Pb.

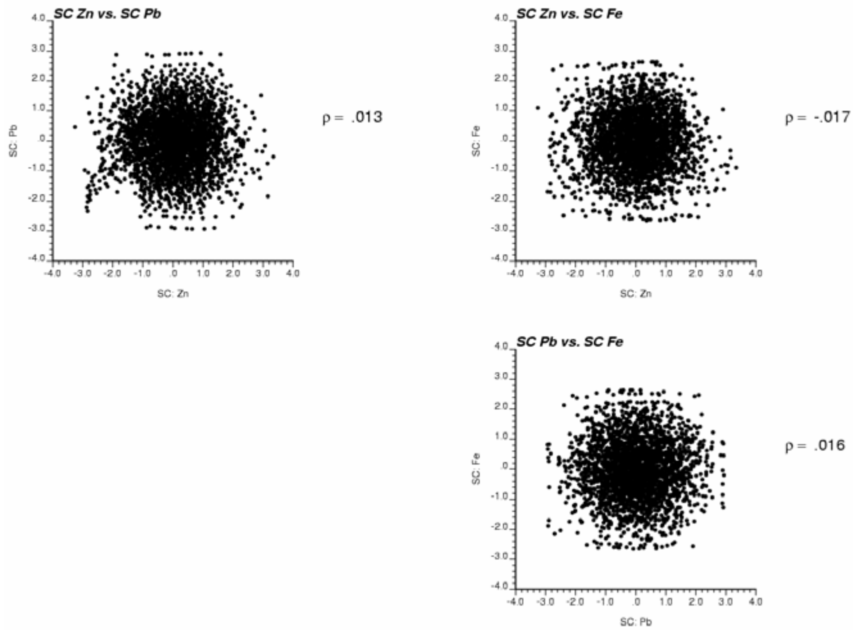

Figure 2 Crossplot between stepwise conditionally transformed variables for Zn, Pb and Fe. Zn was transformed first, then Pb was transformed conditional to Zn, and finally Fe was transformed conditional to both Zn and Pb.

The simulations were thoroughly checked to ensure reproduction of (1) the composite values at their respective locations, (2) the histogram and associated summary statistics, and (3) the variograms in Gaussian space. For this multivariate simulation, the multivariate relations were also checked. The simulated models were upscaled to 25ft×25ft×25ft blocks to facilitate comparisons with the 25ft composites and also the existing long term model.

Figure 3 shows a comparison of the crossplot reproduction from simulation to those crossplots from the 25ft composites and the existing long term resource model. In general, the simulated realizations reproduce the trivariate relations with comparable variability to the 25ft composites; the corresponding plots from the existing long term model shows similar bivariate relations but with noticeably reduced variability.

Once all simulated models were generated and validated on a by rock type basis, a single realization for each variable was obtained by merging the simulated properties from each rock type. With these multiple realizations, the uncertainty at any location and/or region can be assessed (Figure 4).

Validation with Additional Data

Blasthole (BH) data was intentionally excluded from the input data used for model construction. The idea was to assess the predictive ability of the conditional simulation models using the BH data. In particular and due to availability, four variables were compared: Zn, Pb, Fe and Ba.

A rather extensive set of comparisons were performed between: the BH and the DH data, the BH and the E-type estimate of the simulations, the BH and the existing long-term model, and the E-type estimates to the long term model. Table 2 gives the summary of the correlation coefficients from these comparisons. From Table 2, the worst correlations were given by the BH-DH comparison, which was expected given that the paired BH to DH samples may be separated by distances of up to 50ft. This type of comparison was consistent with a comparison between a simulated or E-type model and the BH data, since model values also suffer from being far away from real DH data. The comparison between the BH and both the long term model and the E-type estimate from simulation showed very similar correlations, thus indicating that both models have similar predictive abilities.

Given these comparable results, the conditional simulations were considered an improvement over the existing model in that multivariate relations were honoured. The long term model was generated using independent kriging of each variable. Consequently, there was no assurance to honour the multivariate relations between the different metals. For example, the grade of Zn at any one location had no effect on the modeled grade of Pb, Fe or Ba at that same location. In contrast, the simulations were constructed by explicitly accounting for the multivariate relations between the different variables. As a result, the grade of one variable would affect the simulated value of other metals at the same location.

Comparison of Profit Exercise

In practice, multiple variables are estimated independently with ordinary kriging. This section addresses the impact of the multivariate simulation approach using the stepwise conditional transform relative to the conventional practice of kriging. Note that this exercise is for illustrative purposes only, prices and recovery functions have been synthetically developed and greatly simplified for this specific exercise.

The idea is to compare the profit of ore from both methods with true reference data coming from Red Dog. A profit function is applied to obtain a true profit dataset. A subset of the reference data will be extracted and used to model the grades using both kriging and simulation. The profit function will be applied to these grade models. Based on the expected profit from each approach, each block within the model will be classified as either ore or waste. The true profit at each location is known, so the profit from each model can be calculated.

Profit Function. The real profit function was not available; a simple profit function was developed to account for Zn and Pb grades, recovery functions and metal prices. It was also known that the presence of contaminants such as Ba and Fe would affect Zn recovery, so penalty functions were also considered.

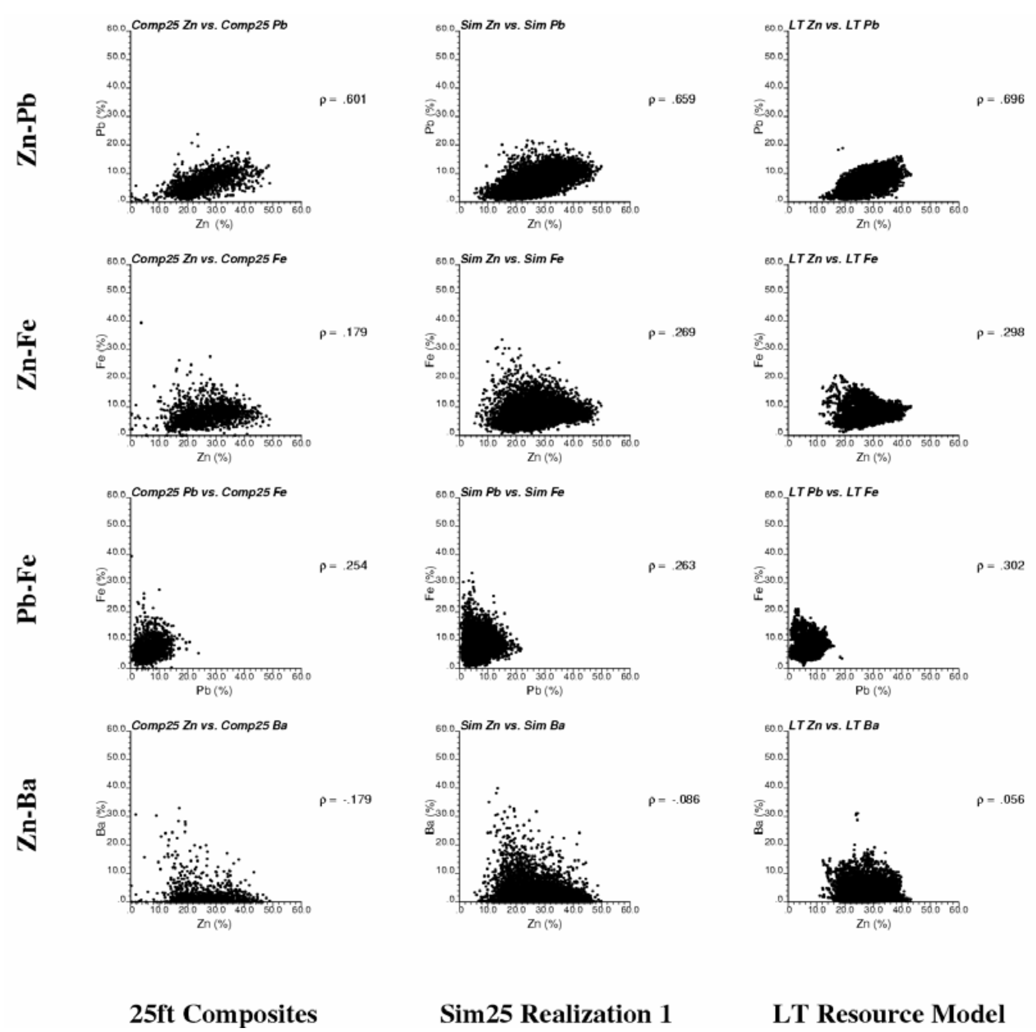

Figure 3 Comparison of multivariate features reproduction for Zn-Pb (top row), Zn-Fe (second row), Pb-Fe (third row), and Zn-Ba (bottom row). Crossplots using the 25ft composites are shown on the left column, from the upscaled simulations are shown in the middle

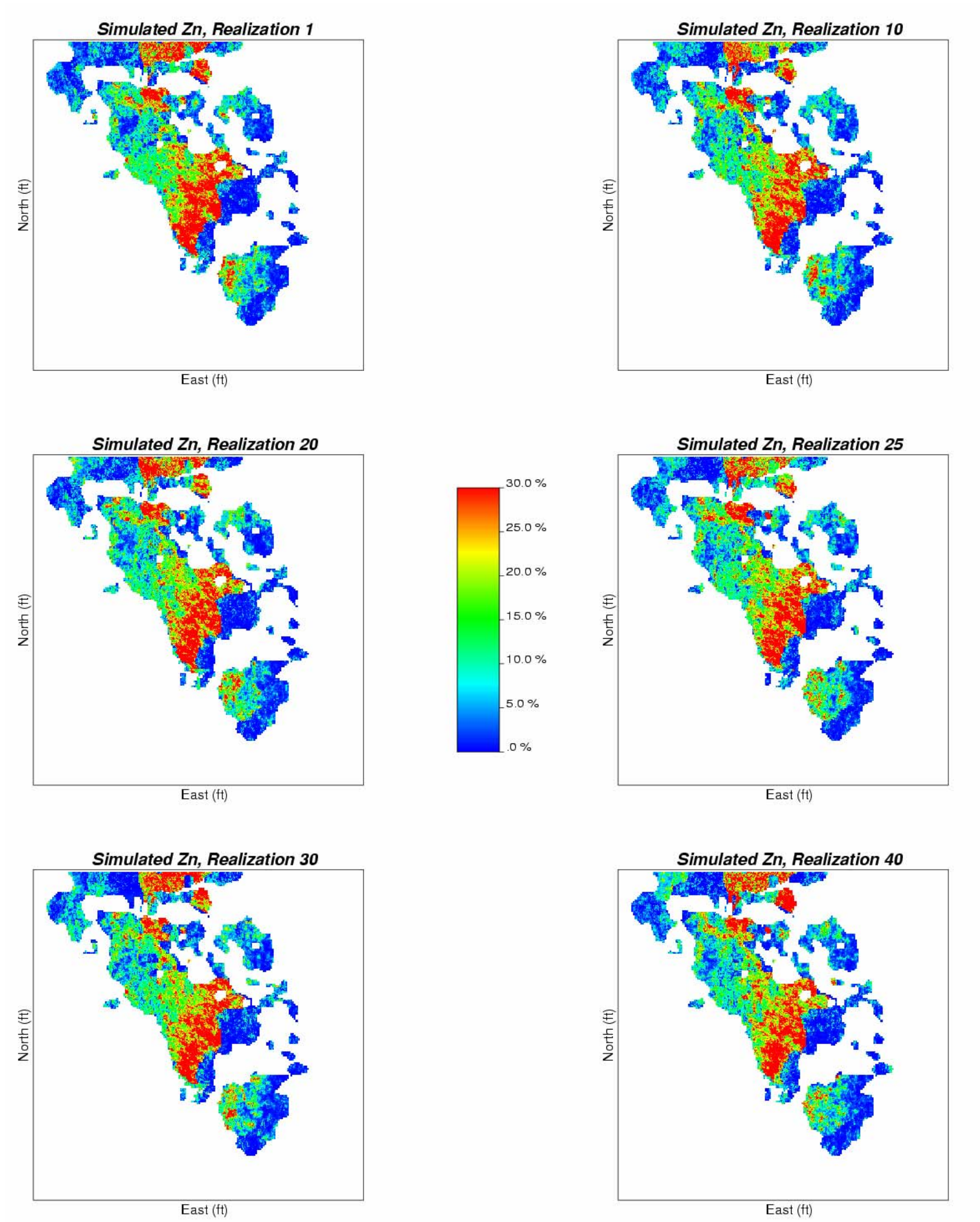

Figure 4 Simulated realizations of Zn at 12.5ft grid resolution.

TABLE 2 Summary of correlation coefficients from all comparisons: BH to DH, BH to Long Term (LT) model, BH to E-type estimate, and Long Term model to E-type estimate.

Variable	BH-DH	BH-LT Model	BH-E-type	LT Model - E-Type
Zn	0.649	0.803	0.802	0.979
Pb	0.563	0.696	0.743	0.894
Fe	0.572	0.641	0.620	0.871
Ba	0.679	0.856	0.858	0.956

Although recovery functions were provided by Teck Cominco, those functions did not actually depend on Ba grade (Leuangthong, 2003). At the time of this work, Teck Cominco was developing new functions based on extensive metallurgical testing. In light of this activity, the recovery functions for Zn, Ba and Fe were developed that consider the effect of contaminant grades as well as some functions provided by Teck Cominco. The Pb recovery is constant.

The constructed models provide the metal grades. All other parameters were developed or chosen to be constant. The metal recoveries for both Zn and Pb were calculated as Red Dog's five year average recovery (1998-2002) based on Teck Cominco's financial report (Teck Cominco, 2003). These were 83.6% Zn recovery and 58.7% Pb recovery. The price for Zn was chosen to be $680/ton of Zn, and the price for Pb was chosen as $380/ton of Pb; both prices were approximated based on the metal prices from the London Metal Exchange in 2003. In order to yield approximately 50% ore and 50% waste classification, the cost per ton mined was chosen arbitrarily.

Reference Data. For a fair comparison to be made, real data must be used. The density and number of BH data available make it an attractive database for true data. Rather than modeling the entire area, only a small area was modeled. The area was chosen to be in a marginal zone, where ore/waste classification based on the models would have the largest impact.

Figure 5 shows the available BH data in the chosen region of 400ft × 400ft in the 850 bench, and the subset of data extracted from this region. The available data consists of 532 BH samples of Zn, Pb, Fe and Ba. From this dataset, 25 samples separated at a nominal 100ft × 100ft spacing were chosen to act as exploration data. This spacing is consistent with the DH data available for Red Dog. This subset of data was used as conditioning data for kriging and simulation.

Model Construction. The model grid was chosen to be 10ft × 10ft × 25ft, which is similar to the 10ft × 12ft × 25ft spacing of the BH data. A total of 1600 blocks were modeled. Further, variograms for both approaches were calculated and fitted using the reference 532 BH data. This filtered out the influence of poor variogram inference due to scarce data.

The variograms for kriging were calculated for the original data, and the variograms for simulation were calculated and fitted for the stepwise conditionally transformed data. In both sets of variograms, a trend was apparent from the experimental points extending beyond the sill of 1.0. This was not surprising given that the area was purposely chosen to be in the transition zone between ore and waste, hence a trend from low to high grades was expected. Trend modeling was not performed for this exercise because of the relatively small area.

For kriging, each variable was estimated independently using ordinary kriging. For simulation, the stepwise conditionally transformed variables were independently simulated using sequential Gaussian simulation to generate 100 realizations of the grades and subsequently back transformed to the original units of the data.

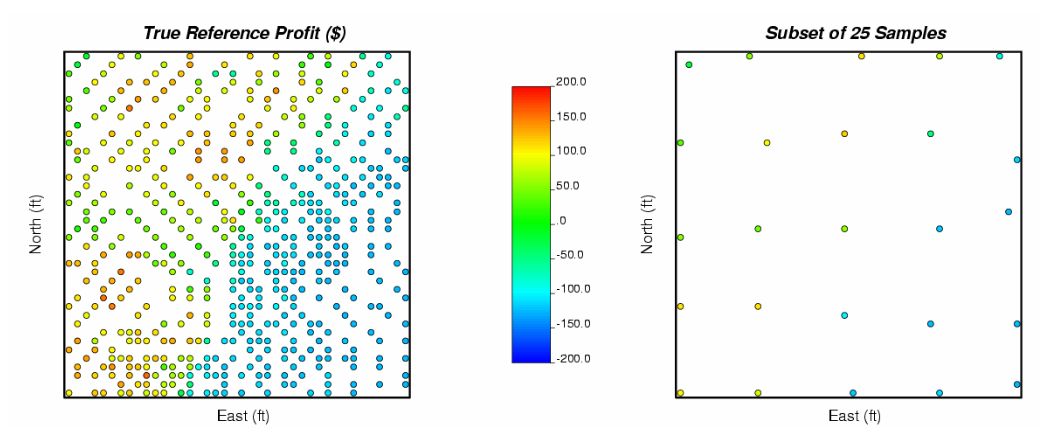

Figure 5 Location map of reference BH data (left) and sampled BH data (right) for use in comparing model approaches.

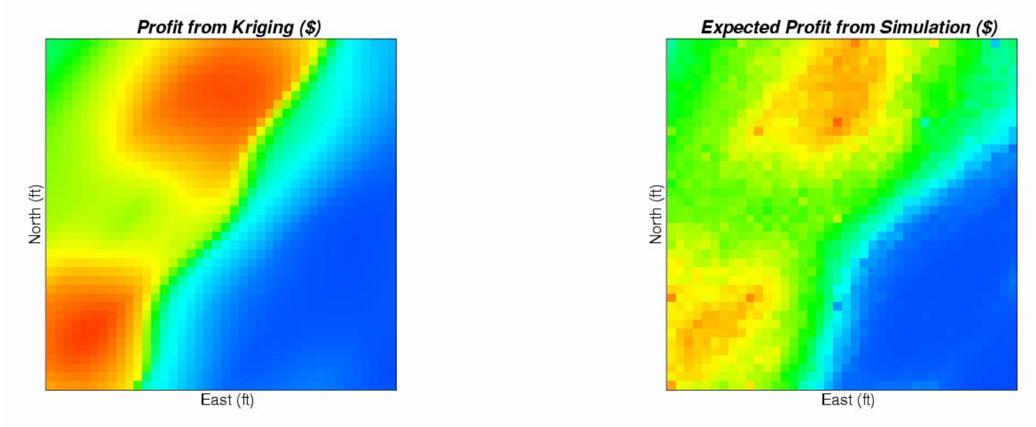

Figure 6 Comparison of true profit map at data locations (top) and the profit map for ore/waste classification from kriging (bottom left) and simulation (bottom right).

Results. These grade models were then processed by applying the profit function at each location within the model. Although 100 realizations of profit were available from simulation, the ore/waste classification was based on the expected profit map obtained by calculating the expected value of profit at each location. Figure 6 shows the profit map obtained from simulation and kriging along with the true profit at the 532 locations where real data were available.

Although 1600 locations were modeled, only the 532 blocks corresponding to locations where true data were available can be compared. At these locations, the true profit was known. The models from kriging and simulation were used to classify the 532 locations as either ore or waste. Figure 7 shows the comparison of the ore/waste classification of the 532 locations from the true reference data to the kriging and the simulation approaches. Overall, both approaches clearly show the waste and ore regions; relatively few blocks were misclassified.

Table 3 shows the summary of the ore/waste classification from both kriging and simulation relative to the true classification. The tables show that the kriging approach resulted in a total 7% of blocks that were misclassified, compared to the 6% misclassified by simulation. From the true profit, 251 blocks (47% of the true data) were classified as ore; simulation correctly classified ore for 98% of those blocks while kriging correctly classified 90% of those blocks.

For those blocks classified as ore, the profit of ore mined as a result of the classification from each method was compared with the true profit of $7.89M. The results from such a comparison showed that the simulation approach yielded $7.28M while kriging yielded $7.06M in profit. Although these profit values appear high for the relatively small area, the relative percentage increase in profit is the key result. Multivariate simulation resulted in 92% of the true profit relative to the 89% yielded by kriging. In practice, this 3% difference may translate to several millions of dollars in increased profit.

TABLE 3 Ore/Waste classification summary of kriging (left) and simulation (right) relative to true ore/waste classification.

		TRUE	
		Ore	Waste
Kriging	Ore	225	11
	Waste	26	270

		TRUE	
		Ore	Waste
Simulation	Ore	246	27
	Waste	5	254

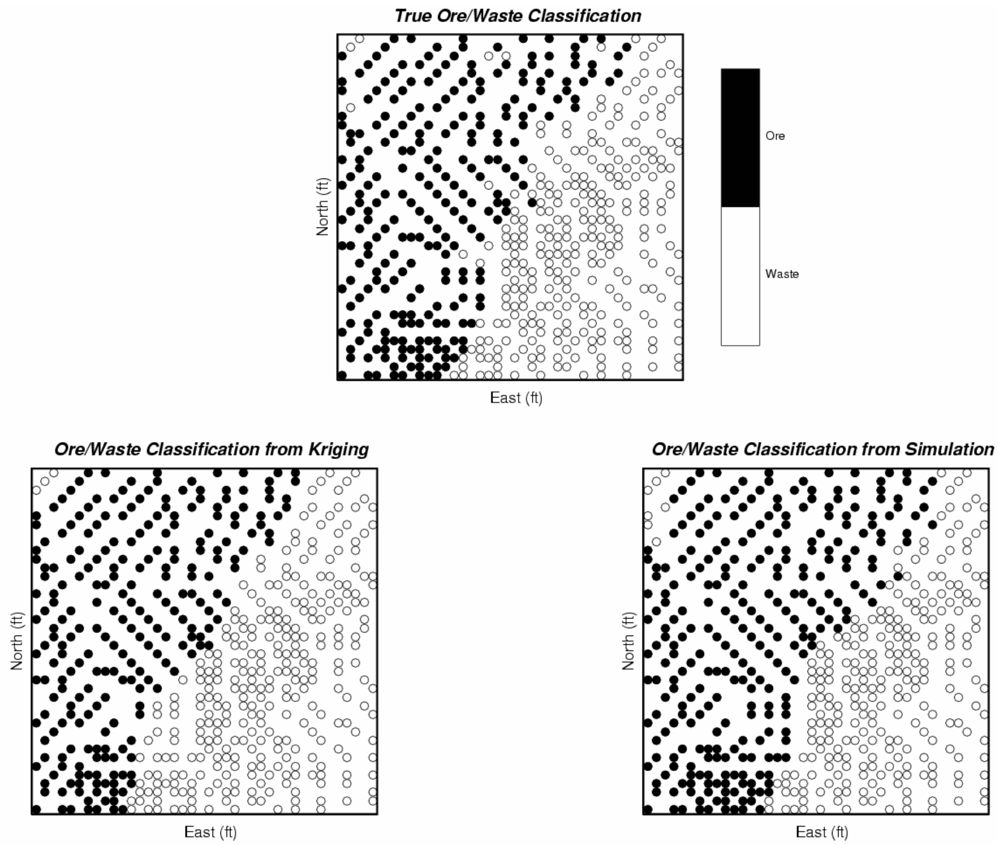

Figure 7 Comparison of true ore/waste classification (top) and the classification from kriging (bottom left) and simulation (bottom right) at data locations.

Conclusions

For the seven variables within the eight rock types, conditional simulation models were constructed using the stepwise conditional transformation technique to account for the multivariate relations. 12.5ft composites and a geology model at 25ft resolution were used to develop these models. Each model was validated by checking reproduction of the input drillhole data, representative histogram, variogram, and the multivariate distributions.

The modeling methodology implemented in this project was quite complex. Conventional approaches are sufficient for straightforward problems; however, for the complexity of the Red Dog deposit, these common approaches are inadequate. The availability of multiple metal grades within multiple rock types warrants some consideration of the relationship between these grades, and how these relationships change from one rock type to the next. The approach documented in this paper was designed to explicitly address this key issue. Consequently, the resulting models not only reproduce the univariate data and its spatial variability, but taken together, they also honour the multivariate relations between the different metals/minerals within the different rock types.

A comparison of the multivariate simulation approach used in this case study and the common practice of kriging multiple variables independently showed that the simulation models resulted in an increase in profit of 3% over the kriging approach, yielding a total of 92% of the true profit.

Acknowledgements

We would like to acknowledge the industry sponsors of the Centre for Computational Geostatistics at the University of Alberta for supporting this research. In particular, thank you to Teck Cominco Limited for the allowing publication of this case study.

References

DEUTSCH, C.V., 1996. Direct assessment of local accuracy and precision. In E.Y. Baafi and N.A. Schofield, ed., Geostatistics Wollongong '96, vol. 1, Holland, Kluwer Academic Publishers, pp. 115-125.

DEUTSCH, C.V., 2003. GSLIB: Geostatistical Software Library and User's Guide. 2nd Edition, Oxford University Press, New York, 376 p.

DEUTSCH, C.V. and JOURNEL, A.G., 1998. GSLIB: Geostatistical Software Library and User's Guide. 2nd Edition, Oxford University Press, New York, 369 p.

ISAAKS, E.H., 1990. The Application of Monte Carlo Methods to the Analysis of Spatially Correlated Data, Ph.D. Thesis, Stanford University, Stanford, CA, 213 p.

JOURNEL, A.G. and XU, W., 1994. Posterior identification of histograms conditional to local data. Mathematical Geology, vol. 26, pp. 323-359.

LEUANGTHONG, O., 2003. Stepwise Conditional Transformation for Multivariate Geostatistical Simulation, Ph.D. Thesis, University of Alberta, Edmonton, 192 p.

LEUANGTHONG, O. and DEUTSCH, C.V., 2003. Stepwise conditional transformation for simulation of multiple variables. Mathematical Geology, vol. 35, pp. 155-173.

TECK COMINCO LIMITED, 2003. Teck Cominco Limited 2002 Annual Report. Retrieved April 2003 from http://www.teckcominco.com/investors/reports/ar2002/tc-2002-operations.pdf.